RUDIMENTS DE COMPUTACIÓ QUÀNTICA

1. Principis i formalismes M.Q.
2. Operadors unitaris
3. Algorisme de Deutsch (1992)

RECORDATORI

Espais de Hilbert

$|v\rangle$ "ket"

$\downarrow$

$\langle v|$ "bra"

	E. vectorials	E. Hilbert
	$\vec{v}+\vec{w}$	$\|v\rangle + \|w\rangle$
	λv	$\lambda\|v\rangle \quad \lambda\in\mathbb{C}$
	$\vec{v}\cdot\vec{w}$	$\langle v\|w\rangle\in\mathbb{C}$
	$\dim(V)<\infty$	$\dim\mathcal{H}\leq\infty$

RECORDATORI

• Espais de Hilbert •

$$\mathcal{H} \xrightarrow{A} \mathcal{H}$$

$$|v\rangle \longrightarrow A|v\rangle$$

A és un operador sii

$$A[|v\rangle + |w\rangle] = A|v\rangle + A|w\rangle$$

A es pot representar per una matriu $n \times n$ en una base

$A^+ \equiv$ transposar elements de la matriu i aplicar el complex conjugat

RECORDATORI

Espais de Hilbert

$A|\phi_i\rangle = \lambda_i|\phi_i\rangle$ M.Q → $A^\dagger = A$ hermítiques

$\lambda_i \in \mathbb{R}$ valors propis

$|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle$ $|\phi_i\rangle$ estats propis de A

$P(\phi_i) = \alpha_i \cdot \alpha_i^*$
$= |\alpha_i|^2$ → la probabilitat d'obtenir estat $|\phi_i\rangle$ amb valor λ_i

$e^{i\beta}|\psi\rangle \cong |\psi\rangle$ → les fases globals no aporten res

$\langle\psi|A|\psi\rangle$ → valor esperat d'un observable

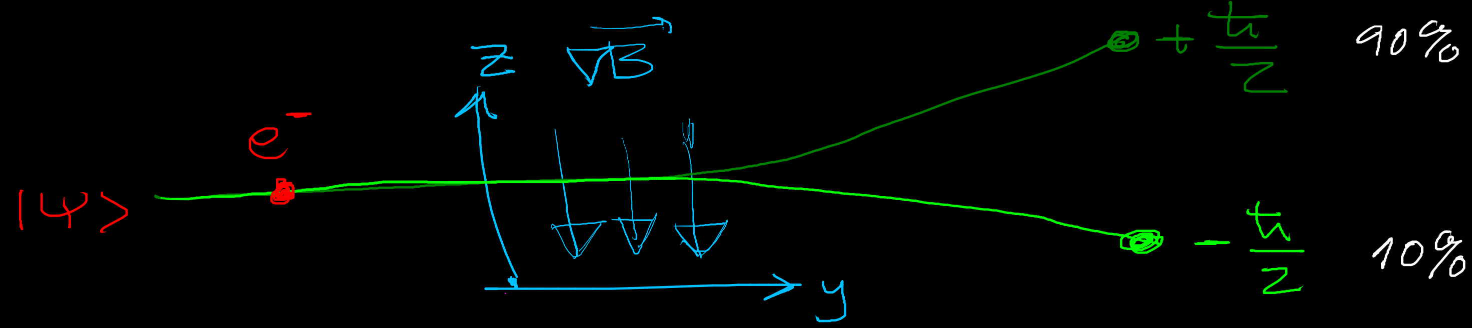
Spin 1/2

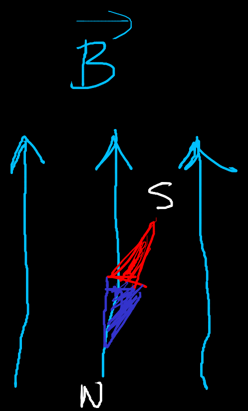
e^- n

μ

p γ

q





$$S_z|+1/2\rangle = \frac{\hbar}{2}|+1/2\rangle$$
$$S_z|-1/2\rangle = -\hbar/2\,|-1/2\rangle$$

$$\hbar = \frac{h}{2\pi}$$

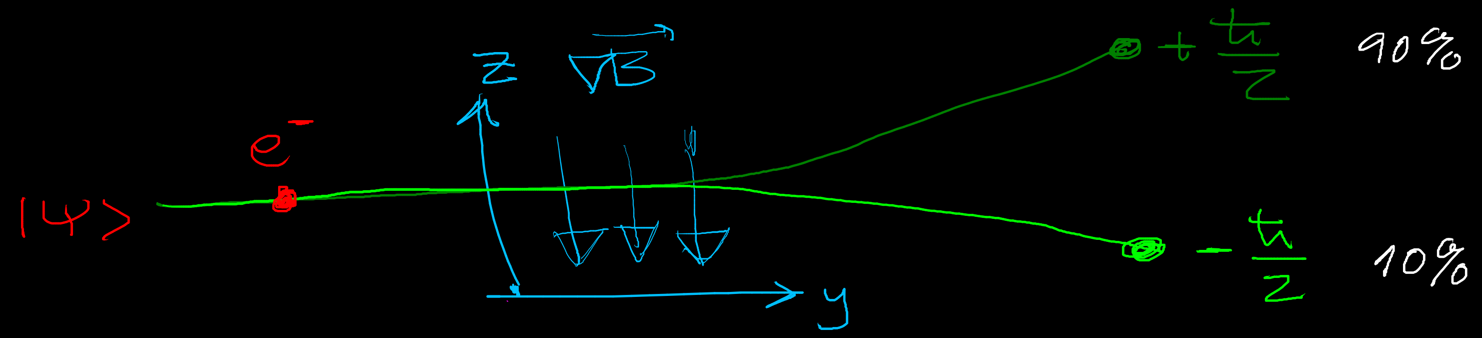
Spin 1/2

e^- n

μ

p ν

q



$$|\psi\rangle = \frac{3i}{\sqrt{10}}|+1/2\rangle + \frac{i}{\sqrt{10}}|-1/2\rangle$$

$$P_{+1/2} = \frac{3i}{\sqrt{10}} \cdot \frac{3(-i)}{\sqrt{10}} = 0{,}9$$

$$P_{-1/2} = \frac{i}{\sqrt{10}} \cdot \frac{-i}{\sqrt{10}} = 0{,}1$$

$$S_z|+1/2\rangle = \frac{\hbar}{2}|+1/2\rangle$$
$$S_z|-1/2\rangle = -\hbar/2\,|-1/2\rangle$$

$$S_x = \frac{\hbar}{2}\underbrace{\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}}_{=\,\sigma_x}, \quad S_y = \frac{\hbar}{2}\underbrace{\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}}_{=\,\sigma_y}, \quad S_z = \frac{\hbar}{2}\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{=\,\sigma_z}$$

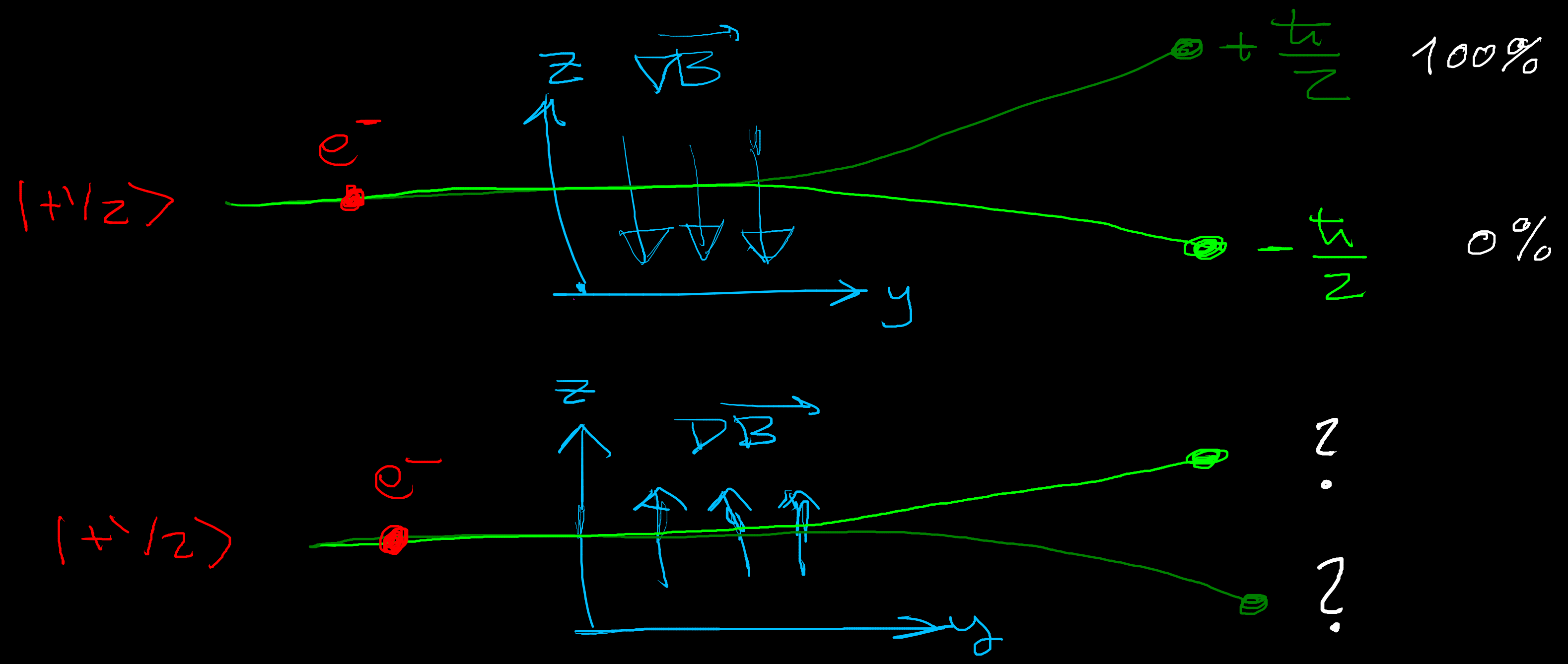
Spin 1/2

e^- n

μ

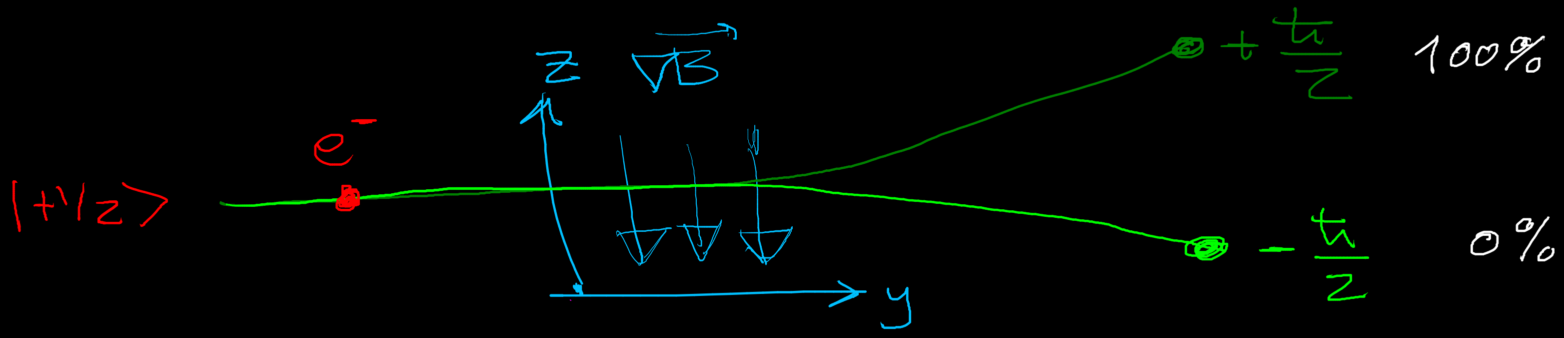
p ν

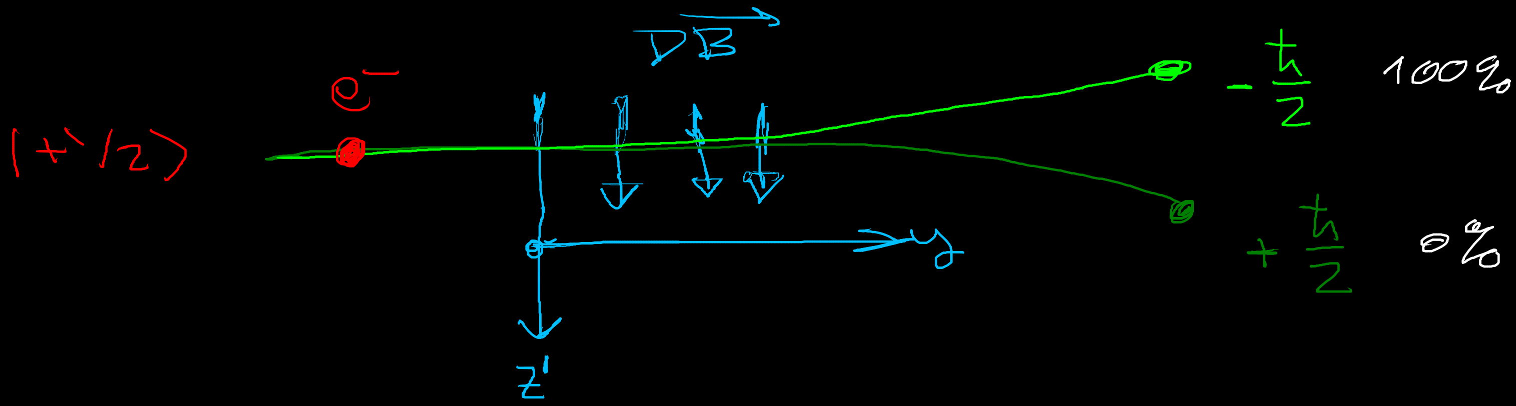
q



Spin 1/2

e^- n μ p ν q





P	z	$-z$
100%	$+\hbar/2$	$-\hbar/2$
0%	$-\frac{\hbar}{2}$	$+\hbar/2$
	$\|+1/2\rangle$	$\|-1/2\rangle$

una aturada en el camí I

Una aturada en el camí I

¿Quant val $e^{0,2}$?

Una aturada en el camí I

Una manera de calcular-ho és sumant coses:

$$= 1{,}221402758\ldots$$

$$\begin{aligned}
e^{0,2} \simeq\ & 1 && = 1 \\
& + 0{,}2 && = 1{,}2 \\
& + \frac{(0{,}2)^2}{2} && = 1{,}22 \\
& + \frac{(0{,}2)^3}{3!} && = 1{,}2213 \\
& + \frac{(0{,}2)^4}{4!} && = 1{,}22137 \\
& \vdots
\end{aligned}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

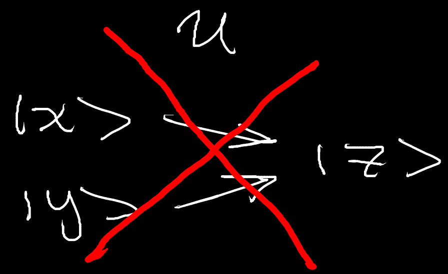
sèrie de Taylor

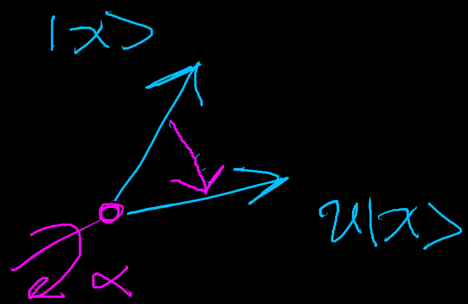
OPERADORS UNITARIS

Un operador $\mathcal{U}$ és unitari sii $\mathcal{U}^+\mathcal{U} = \mathcal{U}\mathcal{U}^+ = I$

P. exemple: $\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

els $\mathcal{U}$ tenen la propietat de ser isometries:

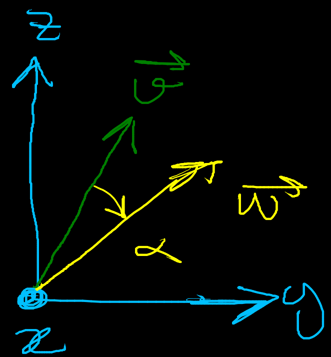




$\mathcal{U} \neq \mathbb{R}$ en general

GRUP SO(3)

SO(3) és el grup de rotacions en el nostre espai $\mathbb{R}^3$



$$e^{\alpha B} \cdot \vec{v} = \vec{w}$$

B es diu generador de la rotació sobre x

$$e^{\alpha B} = I + \frac{(\alpha B)^1}{1!} + \frac{(\alpha B)^2}{2!} + \frac{(\alpha B)^3}{3!} + \dots$$

es demostra que $e^{\alpha B}$ és unitari sii $B^{+} = B$

GRUP SU(2)

Hi ha un conjunt d'operadors unitaris que equivalen a rotacions en l'espai de Hilbert $\mathbb{C}^2$

$$SU(2) = \{ e^{i\alpha B} \mid B^{+} = B, \ \mathrm{tr} B = 0, \ \alpha \in \mathbb{R} \}$$

Si $B = B^{+}$ i $\mathrm{tr} B = 0$ què vol dir?

$$\begin{pmatrix} a & b - ic \\ b + ic & -a \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

GRUP SU(2)

$$\begin{pmatrix} a & b-ic \\ b+ic & -a \end{pmatrix} = a\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{=\sigma_z} + b\underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{=\sigma_x} + c\underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{=\sigma_y}$$

$$B = a\sigma_z + b\sigma_y + c\sigma_y = \alpha\,\hat{n}\cdot\vec{\sigma}$$

ELS OPERADORS D'SPIN GENEREN SU(2)

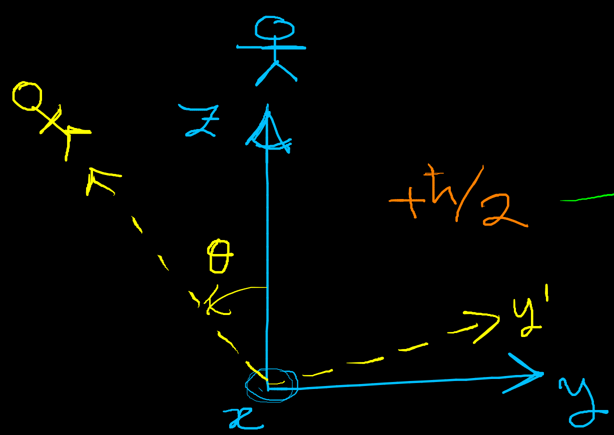
GRUP SU(2)

$$\boxed{e^{i\alpha \hat{n}\cdot\vec{\sigma}} = I\cos\alpha + i\,\hat{n}\cdot\vec{\sigma}\,\sin\alpha}$$

Les rotacions en $\mathbb{C}^2$ estan generades per l'spin $\vec{S}$

$$\boxed{e^{i\alpha \frac{2}{\hbar}\hat{n}\cdot\vec{S}}}$$

$\mathbb{R}^3$ $\qquad\qquad$ $\mathbb{C}^2$



$$|+\tfrac{1}{2}\rangle$$

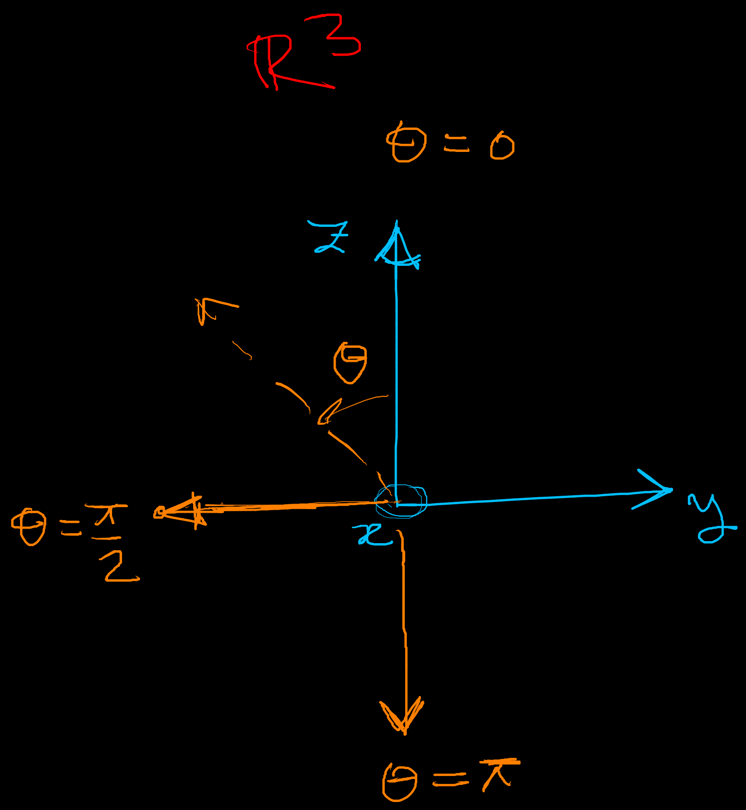
$$e^{i\alpha\frac{2}{\hbar}S_x}|+\tfrac{1}{2}\rangle$$

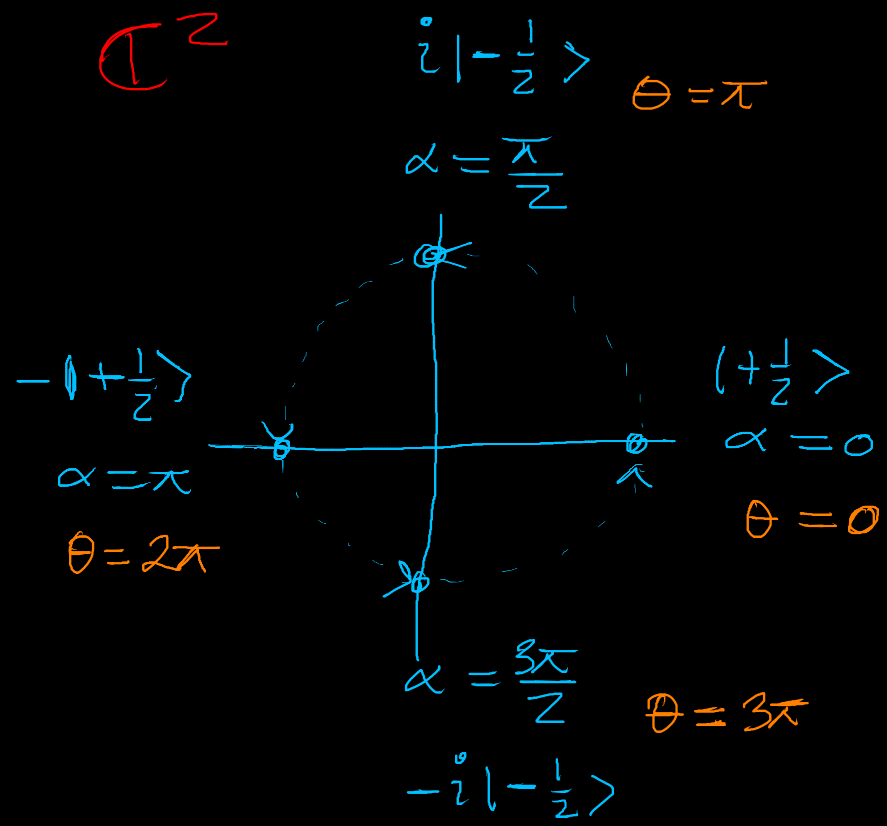
$$\| $$

$$\cos\alpha\,|+\tfrac{1}{2}\rangle + i\sin\alpha\,|-\tfrac{1}{2}\rangle$$

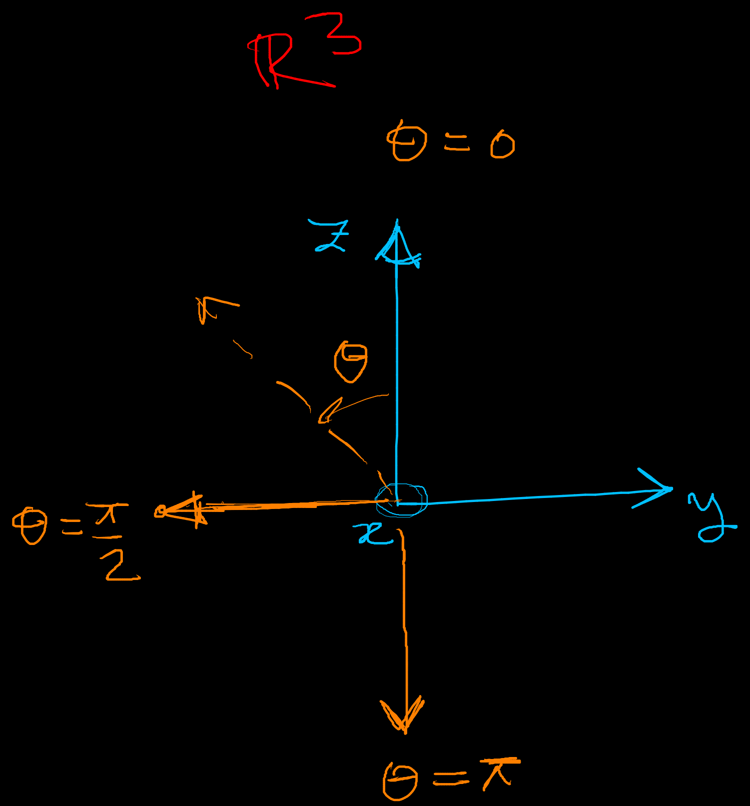
Rotació θ entorn l'eixe x

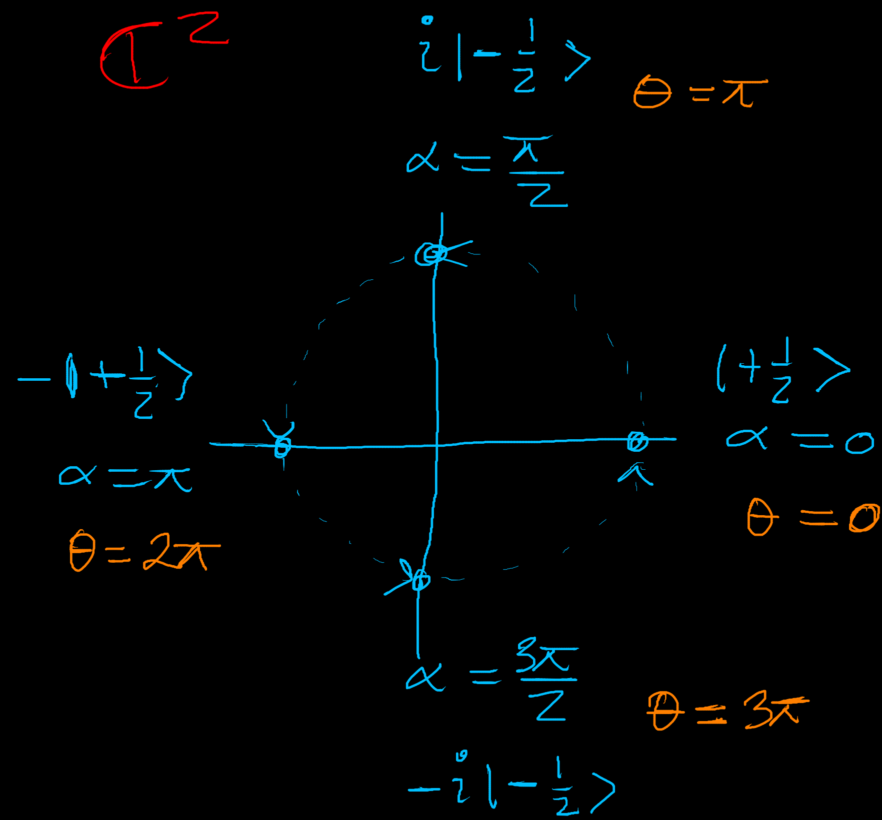
Rotació generada per S_x angle α





$$|R\rangle = \cos\alpha\,|+\tfrac{1}{2}\rangle + i\sin\alpha\,|-\tfrac{1}{2}\rangle$$

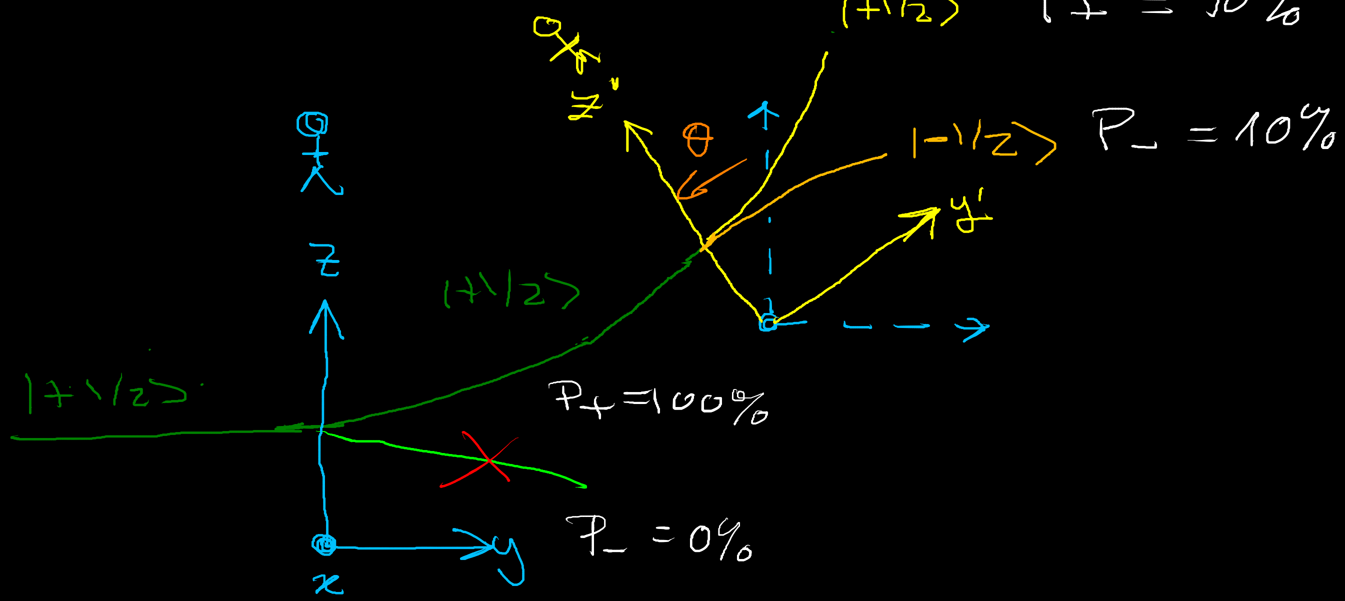


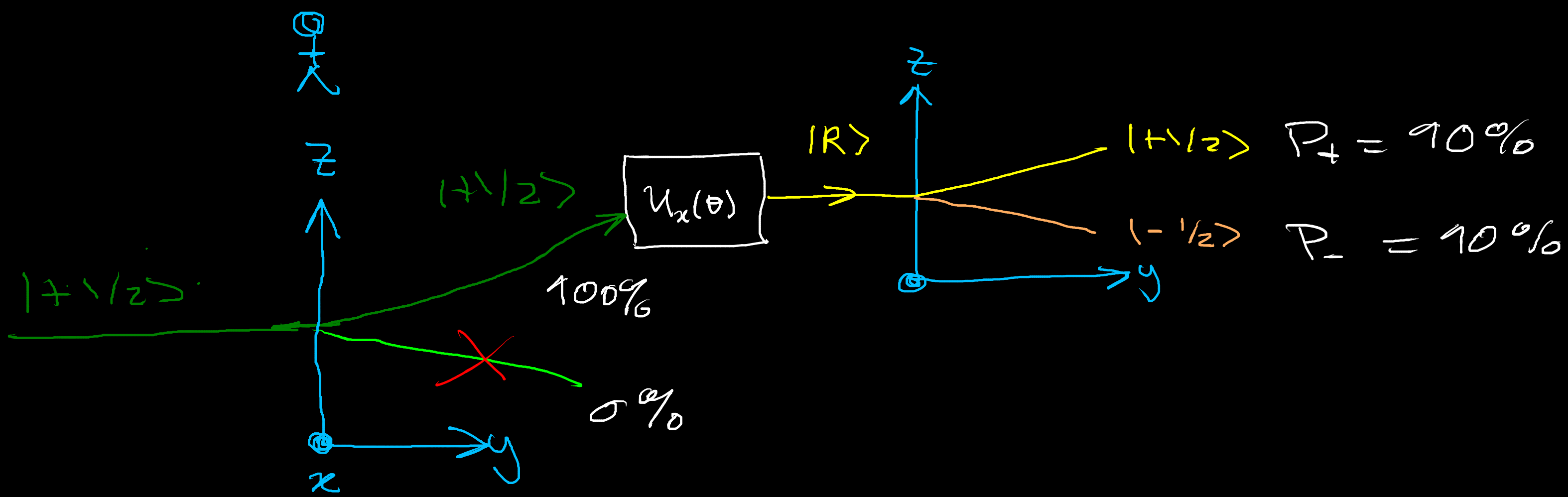


En el món $\mathbb{R}^3$ mesurem $\theta = 2\alpha$ per tant

$$\underbrace{U_x(\theta)|+\tfrac{1}{2}\rangle}_{=\,|R\rangle} = \cos\frac{\theta}{2}|+\tfrac{1}{2}\rangle + i\sin\frac{\theta}{2}|-\tfrac{1}{2}\rangle$$

$(\theta = 37^\circ)$





una aturada en el camí II

COMPUTACIÓ CLÀSSICA

El problema que l'algorisme ha de resoldre:

$$a \longrightarrow \boxed{f} \longrightarrow f(a)$$

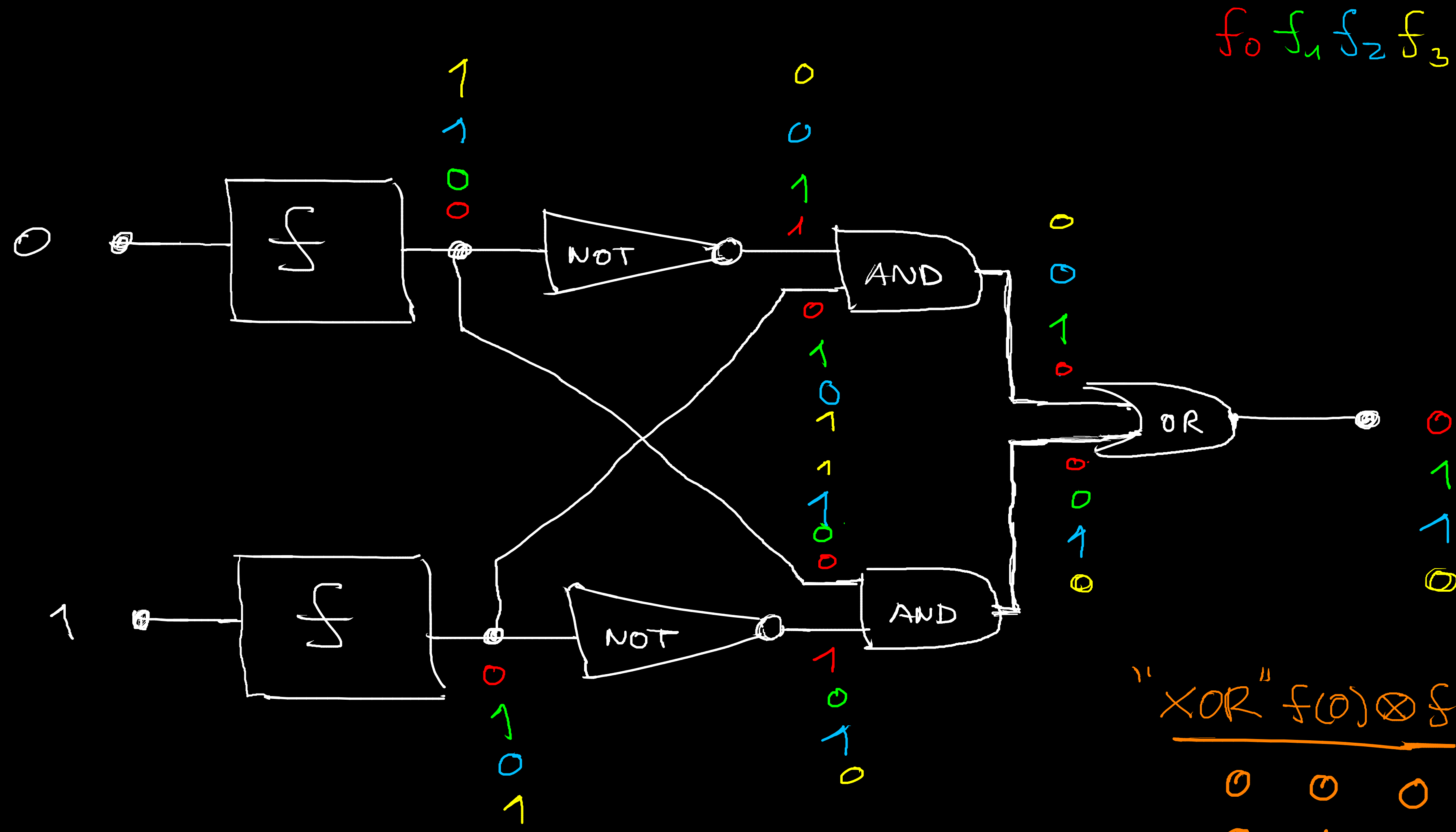
	f_0	f_1	f_2	f_3
0	0	0	1	1
1	0	1	0	1

COMPUTACIÓ CLÀSSICA

f és constant sii la sortida és independent de l'entrada.

ÉS f CONSTANT?

COMPUTACIÓ CLÀSSICA



"XOR" $f(0) \otimes f(1)$

0	0	0
0	1	1
1	0	1
1	1	0

COMPUTACIÓ CLÀSSICA

Si hem de construir un ordinador quàntic cal que compleixi les lleis de la M.Q. inclòs f

ÉS f UNITARI ?

COMPUTACIÓ CLÀSSICA

Si hem de construir un ordinador quàntic cal que compleixi les lleis de la M.Q. inclòs f sino, no pot ser computable quànticament

ÉS f UNITARI ?

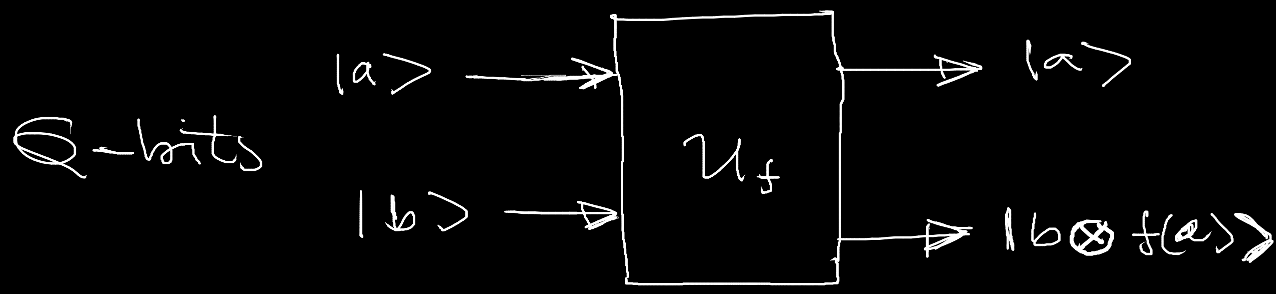
NO! $f_0|0\rangle = |0\rangle$

$f_0|1\rangle = |0\rangle$

COMPUTACIÓ QUÀNTICA

XOR

$0 \otimes 0 = 0$
$0 \otimes 1 = 1$
$1 \otimes 0 = 1$
$1 \otimes 1 = 0$



$$f(0) \otimes f(1) = \begin{cases} 0 & \text{si } f \text{ ct.} \\ 1 & \text{si } f \text{ no ct.} \end{cases}$$

$$|ab\rangle = \{|0\rangle|0\rangle, |0\rangle|1\rangle, |1\rangle|0\rangle, |1\rangle|1\rangle\}$$

P.ex: (f_2)

$$\left.\begin{aligned} U_f|00\rangle &= |0\rangle|0 \otimes f_2(0)\rangle = |01\rangle \\ U_f|01\rangle &= |0\rangle|0 \otimes f_2(0)\rangle = |00\rangle \\ U_f|10\rangle &= |1\rangle|0 \otimes f_2(1)\rangle = |10\rangle \\ U_f|11\rangle &= |1\rangle|1 \otimes f_2(1)\rangle = |11\rangle \end{aligned}\right\} \quad U_{f_2} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

unitària?

SI!

COMPUTACIÓ QUÀNTICA

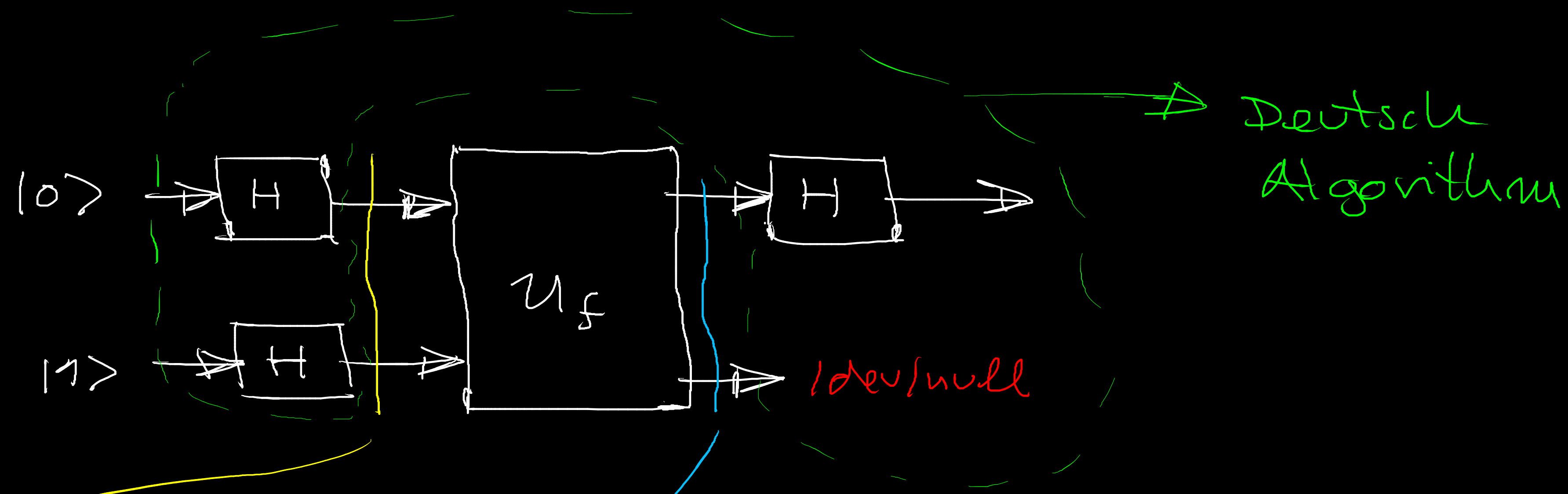
Un operador de Hadamard ve definit així:

$$|a\rangle \longrightarrow \boxed{H} \longrightarrow H|a\rangle$$

$$H = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

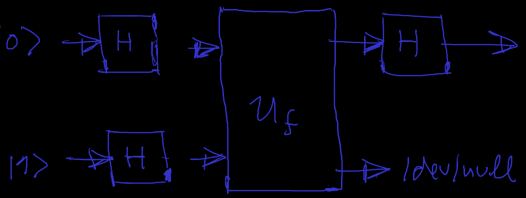
en efecte, també és unitari



$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{2}|0\rangle\left(|0\rangle - |1\rangle\right) + \frac{1}{2}|1\rangle\left(|0\rangle - |1\rangle\right)$$

$$\frac{1}{2}U_f|0\rangle\left(|0\rangle - |1\rangle\right) + \frac{1}{2}U_f|1\rangle\left(|0\rangle - |1\rangle\right)$$

$$= \frac{1}{2}|0\rangle\left(\underbrace{|0\oplus f(0)\rangle - |1\oplus f(0)\rangle}_{= (-1)^{f(0)}(|0\rangle - |1\rangle)}\right) + \frac{1}{2}|1\rangle\left(\underbrace{|0\oplus f(1)\rangle - |1\oplus f(1)\rangle}_{= (-1)^{f(1)}(|0\rangle - |1\rangle)}\right)$$



$$= \frac{1}{2}|0\rangle\left((-1)^{f(0)}(|0\rangle - |1\rangle)\right) + \frac{1}{2}|1\rangle\left((-1)^{f(1)}(|0\rangle - |1\rangle)\right)$$

$$= \frac{1}{2}(-1)^{f(0)}|0\rangle(|0\rangle - |1\rangle) + \frac{1}{2}(-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)$$

$$= \left(\frac{1}{2}(-1)^{f(0)}|0\rangle + \frac{1}{2}(-1)^{f(1)}|1\rangle\right)(|0\rangle - |1\rangle)$$

$$= \underbrace{\left(\frac{1}{\sqrt{2}}(-1)^{f(0)}|0\rangle + \frac{1}{\sqrt{2}}(-1)^{f(1)}|1\rangle\right)}_{1^{er}\ \text{Q-bit}}\underbrace{\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)}_{2^{on}\ \text{Q-bit}}$$

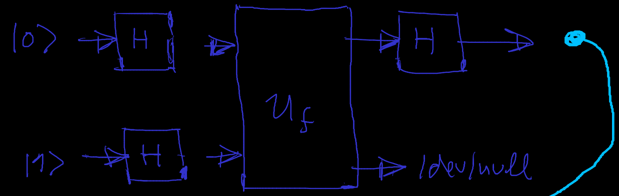
captura el valor de $f(0)$ i $f(1)$ simultàniament!

"phase kick-back"

no ha canviat!

↓

/dev/null



$$(-1)^{f(0)} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2}\,\delta \end{pmatrix} = (-1)^{f(0)} \begin{pmatrix} \frac{1}{2}(1+\delta) \\ \frac{1}{2}(1-\delta) \end{pmatrix}$$

$$(-1)^{f(0)} H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}\underbrace{\frac{(-1)^{f(1)}}{(-1)^{f(0)}}}|1\rangle\right) = (-1)^{f(0)}\left(\frac{1}{2}(1+\delta)|0\rangle + \frac{1}{2}(1-\delta)|1\rangle\right)$$

$$\delta = (-1)^{f(0)\otimes f(1)}$$

si $f(0) \otimes f(1) = 0 \Rightarrow \delta = 1$ $\qquad \cancel{(-1)^{f(0)}}\,|0\rangle$

si $f(0) \otimes f(1) = 1 \Rightarrow \delta = -1$ $\qquad \cancel{(-1)^{f(0)}}\,|1\rangle$

$$\boxed{\begin{array}{l} \text{Si } f \text{ és const.} \Rightarrow |0\rangle \\ \text{Si } f \text{ no és const} \Rightarrow |1\rangle \end{array}}$$